A METHOD FOR THE DIFFERENTIAL MODIFICATION OF THE PROTEIDS IN PERCENTAGE MILK MIXTURES.

By Thompson S. Westcott, M.D.,
INSTRUCTOR IN DISEASES OF CHILDREN, UNIVERSITY OF PENNSYLVANIA.

In a monograph on "The Scientific Modification of Milk," recently published (*International Clinics*, October, 1900), the writer has given numerous formulæ for calculating the proportions of the various milk-derived fluids required in making percentage mixtures. These include combinations of cream and whole milk, cream and separated (fat-free) milk, gravity cream and its separated (under) milk, cream and whey, and simple cream dilutions.

In all these combinations, except indirectly in the all-cream-and-whey mixtures, the total proteids alone are considered in working out the modified proteid percentage, and thus the relative proportions of lactalbumin and caseinogen that are found in the basal milk and cream are not susceptible of variation in these ordinarily employed modifications. In the cream and whey mixtures, which are made by diluting creams of various fat percentage with pure whey, the proportion of lactalbumin is considerably increased as compared with that obtained in the other milk combinations containing the same total proteid percentage; but until recently no means of obtaining definite percentages of lactalbumin and caseinogen has been devised.

A preliminary note upon such a method was communicated by the writer to the Philadelphia Pediatric Society at its meeting in October, 1900. About the same time the problem was undertaken by the Walker-Gordon Laboratory Company, following the experimental work of Franklin W. White and Maynard Ladd, of Boston, who have given a partial exposition of the subject in a valuable paper of recent date (*Philadelphia Medical Journal*, February 2, 1901). In this communication the possibilities of the method are touched upon, but no definite rules for its practical application are suggested. The object of the present paper, therefore, is to describe a simple method for obtaining definite percentages of lactalbumin and caseinogen in combination with definite fat and sugar percentages.

One of the great difficulties in artificial feeding with ordinary milk and cream mixtures lies in the large proportion of caseinogen that must be given to the infant in order to provide a sufficiently rich proportion of the proteid elements. The writer has elsewhere shown[1] that satisfactory nutrition in a child, aged three or four months, cannot be expected from a mixture containing less than 1.50 per cent. of combined proteids, and by a simple calculation it can readily be proven that in the ordinary milk and cream dilutions containing this percentage of proteids, fully 1.20 per cent. consists of caseinogen, just about twice as much as is found in human milk, to say nothing of physical differences in this body as it exists in cow's milk, which render it much less readily digestible by the human suckling than is human caseinogen.

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[1] "The Scientific Modification of Milk," International Clinics, October, 1900, p. 266.

Again, the analysis of human milk proteids shows an average percentage of only 0.59 caseinogen with fully 1.23 lactalbumin (Koenig), while cow's milk contains at least 2.48 per cent. caseinogen with only 0.66 per cent. lactalbumin (Van Slyke). This large excess of lactalbumin over caseinogen in human milk naturally suggests that the former proteid body is much more important for the needs of the growing infant than is the caseinogen, which forms less than one-third the total proteid percentage. It is, therefore, evident that if the analysis of mother's milk is to be used as a standard for the composition of milk mixtures for the artificial feeding of young infants, account must be taken of this wide divergence in the proportions of the two principal proteid bodies in the cow's milk.

Any dilution of cow's milk that brings the caseinogen percentage down to a figure corresponding with that found in human milk (0.59), say one part of milk to four parts of water, reduces at the same time the lactalbumin percentage of the mixture to something like 0.13, about one-tenth of the amount found in the human breast secretion. If, then, lactalbumin must be considered an important element of the breast milk proteids, such a reduction of its percentage in an artificial mixture constitutes a grave defect in the ordinary methods of modification based simply upon a reduction of the total proteid percentage.

Even in a mixture in which the total proteids are made about 1.50 per cent. the percentage of lactalbumin would be only 0.30, a proportion which is still but a fourth of the normal lactalbumin percentage of mother's milk, while, as has already been shown, the caseinogen percentage is still twice as large as the normal.

It is quite possible that in certain cases in which breast milk produces indigestion in the nursing infant some change in the relative proportion of caseinogen may be at fault, and may constitute one of the alterations to which we rather vaguely apply the term "disturbance of equilibrium." If it were possible definitely to modify this percentage for the individual case maternal feeding might be kept up when otherwise it becomes necessary to wean the infant from what in other respects appears to be a desirable maternal supply. This, it must be confessed, is a purely theoretical suggestion upon which no definite analytical investigation has been carried out, but it serves to emphasize the importance of greater flexibility than has been previously afforded for the delicate manipulation of the proteid percentages in artificial feeding, with which at present we are exclusively concerned.

It is thus made manifest that a simple cream, milk, and water mixture, in which total proteids alone are considered, does not fulfil the demand for wider latitude in a regulation of caseinogen and lactalbumin percentages, and that another milk-derived fluid is needed to permit of increase in the deficient lactalbumin percentage, just as cream serves to raise the unduly low percentage of fat resulting from dilution. Such a fluid is whey, which contains fully 1.00 per cent. of proteids other than caseinogen, the latter having been removed from the milk in the process of whey preparation. Whey has always occupied a place in infant diet, but its possibilities have as yet attracted little attention in modern methods of feeding, except in the writings of Kehrer and very recently in those of Monti.[1] Its proteids consist almost entirely of lactalbumin, with a very small proportion of lactoglobulin, and perhaps other soluble proteids in minute quantities. These bodies collectively have been termed the whey-proteids by White and Ladd (*loc. cit.*). The term, however, is rather awkward, and, since lactalbumin forms almost the entire proteid content of the whey, it seems best to speak of the whey-proteids specifically as lactalbumin, and as such the term will be hereafter used.

There is thus presented the problem of combining cream, milk, whey, sugar of milk and a diluent in such proportions as to furnish definite percentages of fat, caseinogen, lactalbumin, and sugar within the limits of practical possibility.

The keynote of such a modification is to be found in the ratio existing in cow's milk between the normal percentages of caseinogen and lactalbumin. According to Koenig's well-known analyses, which give 2.88 per cent. caseinogen to 0.53 lactalbumin, this ratio is seen to be nearly 5½ to 1. White and Ladd (*loc. cit.*) have essayed to establish a new ratio between caseinogen and whey-proteids obtained from the milk of the Walker-Gordon Laboratory of Boston, and in several analyses found that the total proteids averaged 3.84 per cent., of which 0.90 per cent. was whey-proteids and 2.94 per cent. caseinogen. This average of analysis, it will be observed, gives the percentage of total proteids of whole milk considerably higher than that usually obtained, with a percentage of whey-proteids far beyond that of any other analysis with which the writer is familiar in the literature. It is also noteworthy that the percentage of total proteids in the Boston milk is considerably higher than that obtained by Leffmann in Walker-Gordon milk from the Plainsboro dairy, the percentage of which for nine months has averaged about 3.3 (statistics Pediatric Society Milk Commission). The ratio between caseinogen and whey-proteids thus deduced by White and Ladd is therefore about 3 to 1; and it is to be presumed that upon this ratio the laboratory has based the calculations for its so-called whey-cream modifications, which, according to these investigators, are made from 32 per cent. cream, fat-free milk, fat-free whey, and a very concentrated solution of milk sugar.

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[1] Archiv f. Kinderheilk., Bd. xxxi., Heft 1 und 2.

An examination of the literature shows that in 1893 Van Slyke (*Journal of the American Chemical Society*, November, 1893, p. 605) reached the following conclusion: That in analyses of whole milk the caseinogen has varied from 1.93 to 3, and averages 2.48; that the albumin has varied from 0.55 to 0.86, and has averaged 0.66 per cent. This percentage for the lactalbumin also agrees very closely with that given by Blyth (*Foods: Composition and Analysis*, second edition, p. 208), who asserts that the albumin is quite fairly constant and averages 0.70. Van Slyke states that milk rarely contains five parts of casein to one of albumin, and not often does the ratio go above 4½ to 1, and seldom goes below 3 to 1, while the average is somewhat under four parts to one. This proportion also agrees very closely with the estimation previously made by Leeds.

We have thus a rather varied estimation of this important ratio, ranging from Koenig's 5½ to 1, to Van Slyke's and Leeds' 4 to 1, down, finally, to White and Ladd's 3 to 1. Since White and Ladd's ratio of 3 to 1 is based upon what they speak of as "several analyses" only, it would seem more satisfactory for the present to use Van Slyke's average ratio of 4 to 1, which is based upon a considerable number of analyses of milk from American dairies.

A feature of decided interest brought out by White and Ladd's study is the fact that whey contains about 1.00 per cent. proteids, which is slightly higher than Koenig's estimate of 0.86 per cent. This higher figure, too, agrees with analyses published by the United States Department of Agriculture, which, as the mean of a large number of analyses of whey, gave a whey-proteid percentage of 1.00. White and Ladd have suggested that the whey should be made from fat-free milk, and they have thus obtained a fluid containing no fat with 1.00 per cent. of lactalbumin and about 4.8 per cent. sugar. This suggestion is of special value since, in the first place, it decidedly simplifies the somewhat tedious calculations necessitated by the use of a whey containing a small percentage of fat, and, in the second place, it makes for economy in permitting the use of fat-free milk, which in a way is a waste-product of the dairy, and can be obtained from the laboratory at a very moderate cost.

The modification which is now to be described is made from cream (usually 16 per cent., sometimes 20 per cent., and very rarely 32 per cent.), whole milk, fat-free whey, and a diluent, or from various combinations of two or three of these fluids, to which is to be added the amount of sugar in dry form required to bring up the sugar percentage, and, if desired, lime-water to produce the necessary alkalinity. It is quite evident that when all three milk-fluids are combined the fat of the finished mixture will be supplied by the cream and the milk, the caseinogen by the cream and the milk, and the lactalbumin by the cream, by the milk, and, usually in greatest part, by the whey. Sugar is contributed by all three of these milk-fluids, and by the added sugar of milk.

It will be observed that the milk and the cream supply all the caseinogen and a portion of the lactalbumin. The proportion of lactalbumin thus supplied must bear the normal ratio to the amount of caseinogen, or, in other words, the milk and cream supply the desired caseinogen percentage plus a portion of the desired lactalbumin percentage equal to one-fourth of the caseinogen percentage. The balance of the lactalbumin percentage, if any remain to be added, will therefore be furnished by the whey. The practical problem thus becomes quite a simple one.

Calculation of Formulæ. The following symbols will be used in the calculations:

F = the desired fat percentage.
K = " caseinogen per ct.
A = " lactalbumin "
S = " sugar "

C = quantity of cream in ounces.
M = " milk "
Wh = " whey "
L = " dry sugar of milk in ounces.
W = " diluent "
Q = " total mixture "

To these are to be added two intermediate values:

P′ = percentage of combined proteids supplied by milk and cream.
A′ = " of lactalbumin supplied by whey.

The first step is to derive values for P′ and A′. These are readily found, since we know that the milk and cream furnish all the caseinogen percentage plus a part of the lactalbumin percentage equal to one-fourth of the caseinogen percentage; and the percentage of lactalbumin to be furnished by the whey is what remains after deducting from the total desired lactalbumin percentage the percentage already supplied by the milk and cream. We thus find:

$$P' = K + \tfrac{1}{4} K \qquad (1)$$

$$A' = A - \tfrac{1}{4} K \qquad (2)$$

Having found the value of A′, the quantity of whey is very simply calculated by the proportion A′ : 1.00 : : Wh : Q, whence the equation:

$$Wh = \frac{A'}{1.00} \times Q = A' \times Q \qquad (3)$$

The quantities of milk and cream are next found by substituting P′ for P in the regular formulæ :[1]

$$(4) \qquad C = \frac{(F - P')Q}{12.4 \text{ or } 16.8 \text{ or } 29.2}$$

$$(5) \qquad M = \frac{Q \times F}{4} - 4 \text{ or } 5 \text{ or } 8 \times C$$

The quantity of diluent will be found by subtracting from the total quantity of mixture the combined sum of the quantities of cream, milk, and whey, thus:

$$(6) \qquad W = Q - (C + M + Wh)$$

The formula for sugar gives the amount of dry sugar of milk in ounces to be added:

$$(7) \qquad L = \frac{Q \times S - (4\,C + 4.4\,M + 4.8\,Wh)}{100}$$

For the sake of simplicity this formula may be expressed with sufficient accuracy:

$$L = \frac{Q \times S - 4.4\,(C + M + Wh)}{100}$$

In the modifications suggested by White and Ladd a 32 per cent. cream is used in combination with a separated (fat-free) milk. This is quite feasible in the laboratory, but for home modification so rich a cream cannot easily be obtained, and is generally unnecessary when whole milk instead of separated milk is used in combination with it. For the ordinary combinations a 16 per cent. cream will be found sufficiently rich. It is only when the milk formula (5) works out a negative value that a higher fat cream is required, and for the majority of such formulæ a 20 per cent. cream will be found rich enough.

If, however, it is desired to make the modification with fat-free milk instead of whole milk, formulæ (4) and (5) become :[2]

$$(4') \qquad C = \frac{Q \times F}{16 \text{ or } 20 \text{ or } 32}$$

$$(5') \qquad M = \frac{Q \times P' - (3.6 \text{ or } 3.2 \text{ or } 2.8)\,C}{4}$$

In these formulæ, 16 and 3.6, or 20 and 3.2, or 32 and 2.8 are to be taken as 16, 20, or 32 per cent. cream is used.

One or two examples will show how readily these calculations can be made.

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[1] For an explanation of the derivation of formulæ (4), (5), and (7), the reader is referred to the author's monograph on the subject (International Clinics, November, 1900, p. 233). When 16 per cent. cream is used the divisor in (4) is 12.4, and the coefficient of C in (5) is 4. When 20 per cent. cream is required 16.8 and 5 respectively are to be used; and with 32 per cent. cream 29.2 and 8 are similarly to be chosen.

[2] International Clinics, November, 1900, p. 256.

Example I. Required the proportions of whole milk, 16 per cent. cream, whey, and sugar to give a modified formula containing fat, 3.50; caseinogen, 0.80; lactalbumin, 0.50, and sugar, 6.00 in a total mixture of forty ounces.

Here (1) becomes

$$P' = 0.80 + 0.20 = 1.00$$

and (2)

$$A' = 0.50 - 0.20 = 0.30$$

Therefore, we obtain at once from (3)

$$Wh = 0.30 \times 40 = 12 \text{ oz.}$$

By substituting in (4) the value obtained for P′ we get

$$C = \frac{(3.50 - 1.00)\,40}{12.4} = 8 \text{ oz.},$$

and, therefore,

$$M = \frac{3.50 \times 40}{4} - 4 \times 8 = 35 - 32 = 3 \text{ oz.}$$

The quantity of diluent is found by substituting these values in (6):

$$W = 40 - (8 + 3 + 12) = 17 \text{ oz.}$$

The quantity of dry sugar of milk to be dissolved in the diluent before combining the ingredients is found by substituting in (7):

$$L = \frac{40 \times 6 - (4 \times 8 + 4.4 \times 3 + 4.8 \times 12)}{100} = \frac{240 - 102.8}{100} = 1\tfrac{3}{8} \text{ oz.}$$

The modification would, therefore, be made after the following recipe:

Cream (16 per cent.)	8 oz.
Milk	3 "
Whey	12 "
Diluent	17 "
Dry sugar of milk	1⅜ "

If desired, lime-water may be added to produce an alkaline reaction in the proportion of 5 to 10 per cent. of the mixture. Thus two to four ounces of the diluent may be replaced by an equal quantity of lime-water.

If separated milk were used in place of whole milk we should have from (4′) and (5′):

$$C = \frac{40 \times 3.50}{16} = 8\tfrac{3}{4} \text{ oz.}$$

$$M = \frac{40 - 31.5}{4} = 2\tfrac{1}{8} \text{ oz.}$$

The quantity of whey would remain the same as in the previous example, since the value of A′ would not be affected by variations in the values of C and M.

Example II. Required the proportions of 16 per cent. cream, whole milk, whey, and sugar to give a modified formula of forty ounces containing fat, 3.00; caseinogen, 0.40; lactalbumin, 0.75; sugar, 6.50.

Here, as before, we obtain:

$$P' = 0.40 + 0.10 = 0.50$$

$$A' = 0.75 - 0.10 = 0.65$$

Therefore,

$$Wh = 0.65 \times 40 \quad = 26 \text{ oz.}$$

$$C = \frac{(3 - 0.50)\ 40}{12.4} = 8 \text{ oz.}$$

$$M = \frac{3 \times 40}{4} - 4 \times 8 = -2 \text{ oz.}$$

This result shows that a 16 per cent. cream contains too low a proportion of fat, and, therefore, a higher-fat cream must be used. A 20 per cent. cream will serve the purpose, and thus using 16.8 as a divisor in (4) and 5 instead of 4 as a coefficient of C in (5) we obtain:

$$Wh = 0.65 \times 40 = 26 \text{ oz.}$$

$$C \;= \frac{(3 - 0.50)40}{16.8} = 6 \text{ oz.}$$

$$M = \frac{3 \times 40}{4} - 5 \times 6 = 0.$$

$$L = \frac{40 \times 6.50 - 4 \times 6}{100} = 2\tfrac{3}{8} \text{ oz.}$$

and

$$W = 40 - (26 + 6) = 8 \text{ oz.}$$

Practical Limitations of this Modification. It is quite evident that in any mixture containing whey the percentage of lactalbumin could not exceed 1.00, and could never quite reach this figure, since cream or milk added to the mixture to supply fat would have the effect of diluting the whey and thus reducing the lactalbumin percentage below 1.00. This loss could not be entirely made good by the added milk or cream, since the lactalbumin percentage of milk or cream is only one-fifth of its total proteids.

It is therefore important to determine the limits within which caseinogen and lactalbumin percentages may be modified. In the first place, the lowest possible percentage of lactalbumin will be obtained when A' is equal to 0, which is seen to be the case when $A = \frac{1}{4} K$. It is also evident that the lowest possible caseinogen percentage will be obtained when the ratio between fat and caseinogen is such as to require only cream and no milk in the modification. By taking a value for P' in terms of K from (1), we obtain:

$$(8) \qquad P' = \frac{5K}{4}$$

This value of P' may now be substituted for P in a formula representing the relation between P and F in a mixture containing 16 per cent. cream and no milk:[1]

$$P = 0.225\,F, \tag{9}$$

and we obtain
$$\frac{5K}{4} = 0.225\,F,$$

or
$$5K = 0.90\,F;$$

and
$$K = 0.18\,F; \tag{10}$$

and therefore
$$A = 0.045\,F. \tag{11}$$

It thus becomes evident that the lowest possible percentages of caseinogen and lactalbumin are conditioned by the percentage of fat desired. For example, when the fat percentage is 3.00 the lowest caseinogen percentage will be $0.18 \times 3 = 0.54$, and the lowest lactalbumin percentage will be $0.045 \times 3 = 0.135$. Such a formula should work out as a simple cream dilution, without milk or whey; thus, for forty ounces:

$$P' = 0.54 + 0.135 = 0.675$$
$$A' = 0.135 - 0.135 = 0.$$
$$Wh = 0 \times 40 = 0.$$
$$C = \frac{(3 - 0.675)\,40}{12.4} = 7\tfrac{1}{2}\text{ oz.}$$
$$M = \frac{40 \times 3}{4} - 4 \times 7\tfrac{1}{2} = 0.$$

If a 20 per cent. cream be used we may substitute the value of P' from equation (8) in the formula expressing the relation between F and P in 20 per cent. cream dilutions:[2]

$$P = 0.16\,F; \tag{12}$$

and we obtain
$$\frac{5K}{4} = 0.16\,F,$$
$$5K = 0.64\,F,$$

and
$$K = 0.128\,F; \tag{13}$$

and therefore
$$A = 0.032\,F. \tag{14}$$

Thus, for 3 per cent. fat the lowest possible caseinogen percentage from 20 per cent. cream is $0.128 \times 3 = 0.384$, and the lowest lactalbumin percentage is $0.032 \times 3 = 0.096$.

In like manner, for 32 per cent. cream the minimal values for K and A for any percentage of fat are found to be:

$$K = 0.07\,F, \tag{15}$$
$$A = 0.0175\,F; \tag{16}$$

and thus for 3 per cent. of fat the minimum caseinogen percentage is 0.21 and the minimum lactalbumin percentage is 0.0525.

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[1] For derivation of formulæ (9), (12), and (15), the reader is referred to the author's monograph, p. 253.

[2] Ibid.

Starting from such minimum percentages of caseinogen and lactalbumin in simple cream dilutions, increasing values may be obtained as milk and whey are added to the mixture.

The maximum value for caseinogen would be reached when both forms of proteid were furnished by milk without additional cream, the quantity of milk, and consequently the caseinogen percentage, being conditioned as before by the fat percentage. This would obtain when the value of C in formula (4) becomes 0, which requires that

$$F = P'.$$

By substituting here the value of P′ from (8) we obtain:

$$F = \frac{5\,K}{4},$$

or

$$(17) \qquad K = 0.80\ F.$$

This signifies that for any fat percentage the maximum caseinogen percentage is four-fifths as great; thus for 3 per cent. fat the maximum caseinogen percentage is 2.40.

The maximum lactalbumin percentage that may be combined with a maximum caseinogen percentage for any definite fat percentage will be obtained when whey without additional water forms the diluent, the total percentage of caseinogen being furnished by whole milk. This condition is expressed by the equation:

$$Wh = A'Q = Q - M.$$

Referring to formula (6) it will be seen that when C is 0 the equation for milk becomes:

$$M = \frac{QF}{4}.$$

This value for M, substituted above, gives:

$$A'Q = Q - \frac{QF}{4}, \text{ whence by dividing out Q,}$$

$$A' = 1 - \frac{F}{4}; \text{ and by substituting the value of}$$

A′ from (2)

$$A - \frac{K}{4} = 1 - \frac{F}{4},$$

or

$$(18) \qquad A = 1 - \frac{F}{4} + \frac{K}{4},$$

which expresses the maximum value of A for any value of F which determines a maximum value for K. If the value of K in terms of F, as derived from (17), be substituted, we obtain finally:

$$(19) \qquad A = 1 - \frac{F}{4} + \frac{F}{5} = 1 - 0.05\ F.$$

Thus under the conditions of the problem it is evident that the maximum value for the lactalbumin percentage when F is 1.00 and K is a maximum is 0.95; when F is 2.00, it is 0.90; when F is 3.00, it is 0.85, and when F is 4.00, it is 0.80.

It is quite obvious that when K is less than a maximum for any definite fat percentage the lactalbumin percentage may be slightly higher than the figures just given, since a somewhat larger amount of whey could be used to make up for the decrease in the quantity of milk and cream required to give a lower than a maximum caseinogen percentage. The maximum limit of the lactalbumin percentage under these conditions will be attained when all the diluent is whey, which thus is equal to $Q - (M + C)$.

By a similar but considerably more complicated calculation than that by which (19) is derived, we can finally deduce for 16 per cent. cream and whole milk mixtures:

$$A = 1 - 0.0524\,K - 0.008\,F; \tag{20}$$

for 20 per cent. cream,

$$A = 1 - 0.0476\,K - 0.012\,F; \tag{21}$$

and for 32 per cent. cream,

$$A = 1 - 0.0496\,K - 0.010\,F, \tag{22}$$

each of which expresses, for the strength of cream used the maximum value for the lactalbumin percentage for any definite fat and caseinogen percentage. The results obtained by working out these values of A for definite fat values when minimal values for K are used, show that the possible maximal lactalbumin percentage can be increased above that possible for maximal caseinogen values by quantities varying from 0.03 at the lower end of the scale to 0.11 at the upper limit. This, of course, is only possible when there is no watery diluent. Since, however, a small quantity of water is useful in dissolving the additional sugar of milk, which does not dissolve readily in the milky fluids, it is preferable to restrict the lactalbumin percentages to something less than those that can be obtained theoretically. For practical purposes the figures given in the last column of the following table, which are only possible with the minimum caseinogen percentages, may be slightly reduced to those given in the third column as obtainable with maximum caseinogen percentages, and these latter can always be obtained with any caseinogen percentage between the minimum and maximum values. Thus, for 3 per cent. of fat, lactalbumin percentages between 0.06 and 0.85 may be obtained with any possible caseinogen percentage, the limits of which are given in the second column of the table (0.21 to 2.40), provided, always, that the desired lactalbumin percentage shall not fall below one-fourth of the caseinogen percentage.

TABLE SHOWING THE MAXIMUM AND MINIMUM VALUES OF PERCENTAGES OF CASEINOGEN AND LACTALBUMIN OBTAINABLE WITH ANY DEFINITE FAT PERCENTAGE.

Fat percentage.	Range of possible caseinogen percentages.	Range of possible lactalbumin percentages.	Maximum lactalbumin percentage for minimum caseinogen percentage.
1.00 . . .	0.18 (16 per ct.) 0.128 (20 ") 0.07 (32 ") to 0.80	0.045 (16 per ct.) 0.032 (20 ") 0.0175 (32 ") to 0.95	0.98 0.98 0.93
1.25 . . .	0.225 (16 ") 0.16 (20 ") 0.0875 (32 ") to 1.00	0.056 (16 ") 0.040 (20 ") 0.0218 (32 ") to 0.93	0.97 0.97 0.98
1.50 . . .	0.27 (16 ") 0.192 (20 ") 0.105 (32 ") to 1.20	0.0675 (16 ") 0.048 (20 ") 0.0262 (32 ") to 0.92	0.97 0.97 0.97
1.75 . . .	0.315 (16 ") 0.224 (20 ") 0.1225 (32 ") to 1.40	0.0787 (16 ") 0.056 (20 ") 0.0306 (32 ") to 0.91	0.96 0.96 0.97
2.00 . . .	0.36 (16 ") 0.256 (20 ") 0.140 (32 ") to 1.60	0.09 (16 ") 0.064 (20 ") 0.035 (32 ") to 0.90	0.96 0.96 0.97
2.25 . . .	0.405 (16 ") 0.288 (20 ") 0.1575 (32 ") to 1.80	0.10125(16 ") 0.072 (20 ") 0.039 (32 ") to 0.88	0.96 0.95 0.96
2.50 . . .	0.45 (16 ") 0.32 (20 ") 0.175 (32 ") to 2.00	0.1125 (16 ") 0.080 (20 ") 0.04375(32 ") to 0.87	0.95 0.95 0.96
2.75 . . .	0.495 (16 ") 0.352 (20 ") 0.1925 (32 ") to 2.20	0.12375(16 ") 0.088 (20 ") 0.04812(32 ") to 0.86	0.95 0.95 0.96
3.00 . . .	0.54 (16 ") 0.384 (20 ") 0.21 (32 ") to 2.40	0.135 (16 ") 0.096 (20 ") 0.0525 (32 ") to 0.85	0.94 0.94 0.95
3.25 . . .	0.585 (16 ") 0.416 (20 ") 0.2275 (32 ") to 2.60	0.14625(16 ") 0.104 (20 ") 0.05687(32 ") to 0.83	0.94 0.94 0.95
3.50 . . .	0.63 (16 ") 0.448 (20 ") 0.245 (32 ") to 2.80	0.1575 (16 ") 0.112 (20 ") 0.06125(32 ") to 0.82	0.93 0.94 0.95
3.75 . . .	0.675 (16 ") 0.480 (20 ") 0.2625 (32 ") to 3.00	0.16875(16 ") 0.120 (20 ") 0.06565(32 ") to 0.81	0.93 0.93 0.94
4.00 . . .	0.72 (16 ") 0.512 (20 ") 0.28 (32 ") to 3.20	0.18 (16 ") 0.128 (20 ") 0.07 (32 ") to 0.80	0.93 0.92 0.94

A glance at this table will show the limits within which differential modification of proteids is possible. For example, a mixture containing 2.50 fat, 1.00 caseinogen, and 0.50 lactalbumin can be worked out by the formulæ already given, but the same combination of fat and caseinogen with only 0.20 lactalbumin could not be obtained, since the lactalbumin is less than one-fourth of the caseinogen. If 0.25 lactalbumin be chosen, the problem becomes a possible one.

From the various hypotheses just made in working out maximum and minimum values it appears that these mixtures may consist of simple cream dilutions; cream, milk, and water; cream, whey, and water; milk and water; milk, whey, and water; cream and whey; milk and whey; cream, milk, and whey; and cream, milk, whey, and water, any one of which may be required according to the relative percentages of fat, caseinogen, and lactalbumin demanded in the modification. The use of separated (fat-free) milk is also possible in certain cases in combination with cream, with cream and whey, or with cream, whey, and water.

Reasoning conversely, it becomes evident that any of these various combinations of the milk fluids must contain definite percentages of fat, caseinogen, lactalbumin, and sugar; and it is possible to accurately estimate in any mixture of known proportions of these fluids just what the various percentages are. The following formulæ are necessary:

$$\text{(23)}\quad \text{For fat percentage}\begin{cases} \dfrac{C}{Q}\times(16\text{ or }20\text{ or }32)^{*} = \text{fat percentage from cream.}\\[2ex] \dfrac{M}{Q}\times 4 \qquad\qquad\quad = \text{fat percentage from milk.} \end{cases}$$

Sum of these = fat percentage in modification.

$$\text{(24)}\quad \begin{matrix}\text{For caseinogen}\\ \text{percentage}\end{matrix}\begin{cases} \dfrac{C}{Q}\times(3.6\text{ or }3.2\text{ or }2.8)^{*}\times\dfrac{4}{5} = \text{caseinogen per ct. from cream.}\\[2ex] \dfrac{M}{Q}\times 4\times\dfrac{4}{5} = \text{caseinogen percentage from milk.} \end{cases}$$

Sum of these = caseinogen percentage in modification.

$$\text{(25)}\quad \begin{matrix}\text{For lactalbumin}\\ \text{percentage}\end{matrix}\begin{cases} \dfrac{C}{Q}\times(3.6\text{ or }3.2\text{ or }2.8)^{*}\times\dfrac{1}{5} = \text{lactalbumin per ct. from cream.}\\[2ex] \dfrac{M}{Q}\times 4\times\dfrac{1}{5} = \text{lactalbumin percentage from milk.}\\[2ex] \dfrac{Wh}{Q}\times 1 = \text{lactalbumin percentage from whey.} \end{cases}$$

Sum of these = lactalbumin percentage in modification.

A similar calculation would give the sugar percentage of the modification without taking into account the added sugar, or, if this latter be desired, a formula may be derived from (7) by transposition:

$$\text{(26)}\qquad S = \frac{100\,L + 4\,C + 4.4\,M + 4.8\,Wh}{Q}$$

This method of modification, therefore, covers the whole range of modified milk mixtures, and effects a definite modification of fat, caseinogen, lactalbumin, and sugar within possible limits—a very decided advance over any method that hitherto has been proposed either in laboratory or in home modification.

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* One or other of these factors is to be taken as 16 per cent., 20 per cent., or 32 per cent. cream is used, 16 and 3.6, 20 and 3.2, or 32 and 2.8, respectively, being required.

Practical Application of these Modifications. In practice the use of mixtures prepared under this system of modification has yielded the most gratifying results, especially with cream and whey, or milk and whey, or cream, whey, and water, or cream, milk, whey, and water combinations. In the most difficult cases, in which very low percentages of caseinogen are at first required, these combinations may be used in a sort of progressive series, beginning with a cream, whey, and water, or a cream and pure whey mixture. Then by a gradual increase of the caseinogen percentage secured by adding successive small quantities of milk, while the whey or water is decreased equally so as to preserve a constant total quantity of the mixture, the combination becomes one of cream, milk, and whey, or cream, milk, whey, and water; and if the decrease is confined to the whey alone, or if at the time the milk is increased the water is also increased while the whey alone is decreased, we after a time reach a point when the quantity of whey becomes so small, say six to eight ounces, that it may be omitted altogether, its bulk being supplied by the same quantity of water. Thus the mixture becomes a simple cream, milk, and water modification, from which, by continued increase of milk, decrease of water, and, after the fat percentage reaches 4.00, by appropriate decrease of the cream, we finally reach whole milk.

To put this in a concrete example, let us suppose by aid of the table we begin with a formula of fat, 2.00; caseinogen, 0.36; lactalbumin, 0.50 (total proteids, 0.86), and sugar, 5.00. This should require cream, whey, and water, the quantities of which for a forty-ounce mixture will be found to be cream (16 per cent.) five ounces; whey, sixteen and two-fifth ounces; water, eighteen and three-fifth ounces, and sugar of milk, one ounce. If this mixture is found to be well digested an effort may be made to increase the caseinogen percentage. This may be done in one of two ways: either a new formula may be calculated, say for 0.40 per cent. caseinogen, the other percentages being left as before, or a more complex change may be made by adding a small quantity of milk, which will slightly increase not only the caseinogen percentage, but also those of the fat, lactalbumin, and sugar. In general, an increase in all the percentages is desirable, especially when the initial formula has been made low in all its percentages. In the example the addition of a half-ounce of milk, with the decrease of a half-ounce in the quantity of water, will be found to add 0.05 per cent. to the fat, 0.04 per cent. to the caseinogen, 0.055 per cent. to the sugar, and 0.01 per cent. to the lactalbumin. The new mixture will, therefore, contain cream, five ounces; milk, one-half ounce; whey, sixteen and two-fifth ounces; water, eighteen and one-tenth ounces, and sugar of milk, one ounce, and its percentage equivalent will become: F 2.05; K 0.40; A 0.51, and S 5.055.

Of the two plans the latter is much to be preferred, since it permits frequent changes to be made without a recalculation of the quantities, and thus the mother can be given general instructions and can continue to increase the percentages of the formula without constant supervision. Suppose in this way she has gradually added milk and decreased water until her working formula stands: cream, five ounces; milk, ten ounces; whey, sixteen and two-fifth ounces; water, eight and three-fifth ounces, and sugar of milk, one ounce. By means of formulæ (23), (24), and (25) it can be readily calculated that the percentage formula has now become: F 3.00; K 1.16; A 0.70 (total proteids, 1.86), and S 6.10—a very nutritious mixture differing little from that of a good average breast milk, except in the relative proportions of caseinogen and lactalbumin. A closer imitation could be made, if desired, by reducing the percentage of caseinogen and increasing that of lactalbumin, say caseinogen 0.96 and lactalbumin 0.90, which a glance at the table shows to be a possible combination.

In the formula as it now stands (F 3.00; K 1.16; A 0.70, and S 6.10) the lactalbumin percentage contributed by the whey—0.41—is rather too important to be sacrificed at this point, since the total proteid percentage without it would fall to 1.45, and we shall, therefore, continue for a time to increase the milk and decrease the water; or, if it is desirable as quickly as possible to get rid of the necessity of preparing whey, the water may be kept at eight and three-fifth ounces, and the whey alone may be decreased step by step with the increase of the milk.

By this plan the quantity of whey will be decreased while the lactalbumin percentage suffers little change, as can be shown: the increase of an ounce of milk will add 0.10 per cent. to the total proteids (or 0.08 to the caseinogen and 0.02 to the lactalbumin), while the decrease of an ounce in the quantity of whey will take away only 0.025 from the lactalbumin, and thus the total lactalbumin percentage will be scarcely disturbed. Under this plan the addition of eight ounces of milk, with an equal decrease of whey, will add 0.80 per cent. to the total proteids, or 0.64 per cent. caseinogen, and 0.16 per cent. lactalbumin; while the decrease of eight ounces of whey will subtract 0.20 per cent. lactalbumin, leaving the total lactalbumin at 0.66 per cent., only 0.04 below the previous formula. At this point the remaining eight and two-fifth ounces of whey, which contribute only 0.21 per cent. lactalbumin, may be omitted, its bulk being replaced by simple diluent. We have thus reached by gradual steps a cream, milk, and water modification of the ordinary kind, consisting of cream, five ounces; milk, eighteen ounces; water, seventeen ounces, and sugar, one ounce, containing these percentages: F 3.80; K 1.80; A 0.45 (total proteids, 2.25); and S about 5.00, the withdrawal of the whey, which is rich in sugar, having decreased the sugar percentage from 6.10. If so desired this percentage may be increased by the addition of a quantity of sugar of milk to be determined by formula (7).

Starting, now, from the cream, milk, and water combination, containing cream, five ounces; milk, eighteen ounces, and water, seventeen ounces, the milk may be gradually increased until two ounces more are added to the milk and two ounces taken from the water. At this point the fat percentage becomes 4.00, which is as high as it is generally desirable to have it. Subsequent increase of the milk may be associated with a decrease of the cream, one drachm of 16 per cent. cream being omitted for each half-ounce of milk added, or one drachm of 20 per cent. cream for five drachms of milk. By this device the fat is kept at about 4 per cent., while the proteid percentage goes on increasing until, when finally the quantity of milk reaches forty ounces, the cream and water have been replaced entirely by milk and the mixture becomes unmodified whole milk.

This example shows one of many possible methods of varying the relative proportions of caseinogen and lactalbumin in feeding a single case. If evidences of casein indigestion should appear in the stools the caseinogen percentage may be decreased, while the fat and lactalbumin percentages are varied according to indications. For example, by reference to the table, it will be seen that for 2 per cent. fat a caseinogen percentage as low as 0.14 may be obtained by using a 32 per cent. cream, and that even with a 16 per cent. cream the caseinogen percentage may be kept as low as 0.36. With this latter very low caseinogen percentage any lactalbumin percentage from 0.09 up to 0.96 per cent. may be obtained, a resulting total of 1.32 per cent. of mixed proteids for the maximum lactalbumin percentage. With a higher percentage of fat, say 3.00, the caseinogen percentage may still be kept as low as 0.21 if a 32 per cent. cream be used; and even with a 16 per cent. cream the percentage need not be higher than 0.54.

In the severer cases of chronic gastric catarrh it will usually be found by observation that a low percentage of fat gives better results than a medium or high percentage (2.50 to 4.00), and in such cases a combination of milk and whey at first may be much better borne by the stomach than a cream and whey, or a cream, milk, and whey mixture. In such a modification the fat percentage and the caseinogen percentage are nearly equal, the latter being four-fifths of the former, as is indicated by formula (17); and, as larger quantities of milk are added, this same ratio of fat to caseinogen is maintained, the percentages being kept under control by means of formulæ (23) and (24). After a time the fat may be increased more rapidly, if desired, by calculating out the quantities for any chosen percentage combination by means of the regular formulæ (1), (2), (3), (4), and (5).

Partial Predigestion. While digestive equilibrium can be secured by a sufficient reduction in the percentages of the proteids and fat, it must not be overlooked that an artificial mixture containing less than 1.50 proteids does not furnish what has been shown to be a satisfactory living ration for any but the youngest infant. It is true that life can be supported indefinitely upon a mixture containing a proteid percentage something more than 1.00, but, with this, satisfactory gain in weight and fibre cannot be expected after the first few weeks of life. A percentage of 1.50 for the proteids must, therefore, be secured just as soon as the infant's digestive powers will permit of it, and to this end partial predigestion of the mixture will give material aid. For example, an infant of four weeks who failed to digest satisfactorily a mixture containing 2.88 fat and 0.63 total proteids was enabled by partial peptonization perfectly to assimilate a mixture in which the total proteids were raised to 1.00. Such a gain in the proportion of proteids that can be assimilated fully justifies the resort to this expedient. In the more difficult cases partial predigestion of the cream and milk is often a very valuable help. Here the cream and milk, together with a portion of the diluent about equal in quantity to that of the cream and milk combined, may be partially predigested for eight, ten, twelve, or fifteen minutes at a temperature of 105° F., after which the mixture is rapidly raised to a temperature of 150° F. to check further action of the pancreatin, and then cooled before the addition of the whey. Or, the whey may be added to the warm milk and cream mixture after peptonization has been carried on for five to seven minutes, and the process then continued for five to eight minutes longer, after which rapid heating to 150°.

Preparation of the Mixtures. There is no question that the best and most accurate results are obtainable when the ingredients of the mixture are prepared and combined in the laboratory. If for various reasons this is impracticable, perfectly satisfactory results may be secured by home modification in the hands of an intelligent mother with materials supplied by the laboratory.

Even in the absence of laboratory help very satisfactory results can be obtained by applying these principles of computation to the best materials that come to hand. While there will be divergences between the actual and theoretical percentages in the resulting mixture, the same flexibility of variation will be afforded, and relative increase and decrease in the initial percentages can always be effected, according to the demands of the case. The most important prerequisite is a cream of fairly constant fat percentage. This being given, the proteid percentages may be reckoned on the theoretical basis with comparative accuracy.

For most of the modifications that will be used under these conditions two quart jars of whole milk a day will be sufficient, one to supply the cream and the separated milk for making the whey, the other to supply the whole milk and to make up any deficiencies in the quantities of cream and whey obtained from the first bottle. Many specimens of bottled milk show a considerable rise of cream even when first delivered, and a few hours of standing in ice or ice and salt will add considerably to the separation, which can be clearly seen at the top of the bottle. By means of a glass siphon the lower separated milk may be drawn off and kept for making the whey, while the upper creamy layer will be left in the bottle to be used as the cream. Whey made from this separated milk, while not entirely free from fat, will contain only a very minute percentage of it.

The cream remaining in the bottle should be thoroughly stirred to obtain uniformity of composition, and samples of this should be taken for several days and analyzed for the fat percentage, an average of which should be taken for the fat percentage of the cream to be used in the modification. If this be impracticable it will be safe to assume for the purposes of calculation that it averages 16 per cent. After all, in any given case the actual percentage of fat in the mixture is not so important as the relative changes that may be made. If the infant can digest a certain proportion of fat, it does not matter whether it actually be 2.00 per cent. or 2.25 per cent. A relative increase or decrease of 0.25 per cent., for instance, would raise or lower either percentage by about the same amount, and effect the desired change for this particular baby. The only disadvantage in dealing with uncertainties in the actual percentages of the component milk fluids of a mixture is that the physician cannot so sharply crystallize his experience in percentage feeding as to gain an accurate estimate of the significance of definite percentages as applied to digestive capacities.

In preparing the whey the separated milk is warmed to a temperature of 100° F., and essence of pepsin, liquid rennet, or a junket tablet added to it. About two teaspoonfuls of essence of pepsin to the pint are required, while liquid rennet, according to White and Ladd, is active in the proportion of one drachm to two quarts. The junket tablet is a cheap, convenient, and very efficient means of precipitating the curd, and has been generally preferred by mothers who have tried all three agents.

The milk is kept at 100° F. until coagulation takes place, and it is then put aside in the refrigerator to favor contraction of the curd and exudation of the whey. At the end of half an hour the solid curd is to be cut into small cubes, put into a freshly-boiled cheese-cloth bag of double thickness, and allowed to drain simply by gravity, since squeezing forces some of the curd through the meshes of the bag. After the whey is thoroughly drained it should be heated to a temperature of 150° F. in order to destroy the rennin, and again cooled before being finally mixed with the other fluids.[1] When partial predigestion of the milk and cream has been effected this heating of the whey is not necessary, unless it be desired for its Pasteurizing action. Otherwise, in combination with raw milk the excess of rennin in the whey would cause precipitation of fresh curd.

All that now remains is to mix the cream, whey, and whole milk with the diluent in which the milk sugar has been previously dissolved, divide the mixture into bottles each containing the amount for one feeding, and put away in the refrigerator.

In conclusion, it may be stated that differential modification of the proteids is not offered as a panacea for all the difficulties of infant feeding. In some cases even with the most carefully thought-out combinations disappointment must be expected. In a very few cases the addition of whey to the mixture in any proportion has seemed to produce pain and indigestion. That such infants became comfortable under the use of a 1 in 32 dilution of condensed milk probably indicates that the power of digesting proteids was very weak, since this dilution of condensed milk contains only about 0.26 per cent. of proteids, a proportion considerably below any of the total percentages tried in the differential modifications. It is evident, therefore, that in some cases whey does not seem to be well borne. These few failures, however, in a rather extensive experience up to the present time, simply serve as the exceptions that prove the general rule.

In a large number of other infants, of from four to five months or older, most of whom suffered from serious catarrhal disease, the result of previous faulty methods of feeding, whey has been perfectly borne, and has yielded results which seemed impossible under other forms of feeding. In not a few of these cases whey has formed the almost exclusive means of supplying proteids in mixtures consisting of cream and all whey. In all these cases the improvement of nutrition under whey modifications has been striking and comparable only to that to be sometimes attained by resort to the breast milk of a perfectly satisfactory wet-nurse.

After all, the last word upon infant feeding still remains to be written; but it is confidently claimed that differential modification has brought us several steps nearer to the hoped-for goal. Further study of its remarkable possibilities may yet explain and correct the few failures that occasionally occur.

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[1] This temperature, 150° F., is lower than the writer has elsewhere recommended (170° F.), but according to White and Ladd's experiments, the lower temperature is sufficient to destroy the rennin and does not injure the whey by coagulating a portion of its lactalbumin, a change which begins, as Hammarsten has shown, at a temperature somewhat below 170° F.